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Nuclear Structure Calculations using Different Symmetries of ^{20}Ne nucleus

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Abstract— *Using the two forms of Fish-Bone potential (I and II), a self-consistent calculations are carried out to perform the analysis of binding energies, root mean square radii and form factors using different configuration symmetries of ^{20}Ne nucleus. A computer simulation search program has been introduced to solve this problem. The Hilbert space was restricted to three and four dimensional variational function space spanned by single spherical harmonic oscillator orbits. A comparison using T_d and D_{3h} configuration symmetries are carried out.*
Keywords— Alpha Clustering / Configuration Symmetry /Fish-Bone potential.

I. INTRODUCTION

The concept of cluster structure in nuclei has been a subject of interest since the early days of nuclear physics till now [1]-[3]. The idea is largely supported by the fact that alpha particles have exceptional stability and that they are the heaviest particles emitted in natural radioactivity. Various models have been proposed to account for possible nuclear clustering and to study its effects [4]. Later, a simple version of the model, in which the alphas are assumed to have no internal structure is considered. The concept of alpha-clustering has found many applications to nuclear reactions and nuclear structures [5,6] .

Many such clusters are possible in principle, but the formation probability depends on the stability of the cluster, and of all possible clusters the alpha particle is the most stable due to its high symmetry and binding energy. Thus a discussion of clustering in nuclei is mainly confined to alpha-particle clustering. For the α-nuclei ^{8}Be, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, the geometrical equilibrium configuration of the α-particles are generally assumed to belong to the symmetry point groups $D_{\infty h}$, D_{3h}, T_d, D_{3h}, O_h (or alternatively D_{4h}) , D_{5h}, D_{6h}, respectively. The ^{20}Ne have also two other configuration symmetries; the T_d configuration symmetry and the D_{2d} configuration symmetry in addition to the D_{3h} configuration symmetry.

In each configuration studied, it was assumed that the nucleons form persistent alpha-particle clusters arranged in some symmetric fashion.

The Fish-Bone potential [7] of composite particles simulates the Pauli effect by nonlocal terms. The α-α fishbone potential be determined by simultaneously fitting to two-α resonance energies, experimental phase shifts, and three-α binding energies. It was found that, essentially, a simple Gaussian can provide a good description of two-α and three-α experimental data without invoking three-body potentials. Many authors adopted the fishbone model because, in their opinion, this is the most elaborated phenomenological cluster-model-motivated potential. The variant of the fishbone potential has been designed to minimize and to

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neglect the three-body potential. Therefore, they can try to determine the interaction by a simultaneous fit to two- and three-body data [8]·

The aim of this work is to employ the D_{3h} configuration symmetry and the T_d configuration symmetry of ^{20}Ne nucleus and using the Fish-Bone potential of type I and II to obtain the binding energies, root mean square radius, and the form factors of the ^{20}Ne nucleus.

II. THE THEORY

We consider a system of N identical bosons described by a Hamiltonian of the usual form

$$H = \hat{T} + \hat{V} = \sum_{i=1}^{N} t(i) + \sum_{i \neq j=1}^{N} v(i,j) \tag{1}$$

Where t(i) is the kinetic energy operator i^{th} particle and v(i,j) the two-body interaction. In Hartree-Fock method, one takes for the best choice of the normalized wave function ψ the one that it minimizes the expectation value of the Hamiltonian H

$$\delta \langle \psi | H | \psi \rangle = 0 \tag{2}$$

In most Hatree-Fock calculations for light nuclei one has taken the subspace spanned by the lowest harmonic oscillator shell l j >. We assume that all the particles occupy the same orbital λ belonging to the average field. Hence the intrinsic state of the whole system would be described by the symmetric wave function

In this subspace, the HF orbitals l λ> are then determined by their expansion coefficients m^{λ}_{j} .

$$\psi(1,2,\ldots\ldots,N) = \lambda(1)\,\lambda(2)\,\lambda(3)\ldots\ldots\ldots\lambda(N) \tag{3}$$

$$|\lambda\rangle = \sum_{j=1}^{N} m_j^{\lambda} |j\rangle \tag{4}$$

And the HF-Hamiltonian h(λ_1 , λ_2 ,….. λ_N) is replaced by the matrix

$$\langle i|h|j\rangle = \langle i|t|j\rangle + \sum_{\lambda}\sum_{k}\sum_{l} m_k^{*\lambda} \langle i\,k|v|j\,l\rangle_s\, m_l^{\lambda} \tag{5}$$

Where

$$\langle i\,k|v|j\,l\rangle_s = \langle i\,k|v|j\,l\rangle + \langle i\,k|v|l\,j\rangle \tag{6}$$

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The HF equations are then replaced by the matrix equation

$$\sum_j \langle i|h|j\rangle m_j^{\lambda} = \varepsilon\, m_i^{\lambda} \tag{7}$$

One proceeds by iteration until self-consistency is achieved

Fish-Bone potential:

A fishbone potential of the α-α system was determined by Schmid E. W. [9] **.** The harmonic oscillator width parameter was fixed to $a = 0.55$ fm^{-2} and the local potential was taken in the form

$$V_l(r) = v_0\, exp(-\beta r^2) + \frac{4e^2}{r} erf\left(\sqrt{\frac{2a}{3}}\, r\right) \tag{8}$$

where v_0 = -108.41998MeV and β = 0.18898fm^{-2} and called Fish-Bone I(FB-1)While this potential provides a reasonably good fit to $l = 0$ and $l = 2$ and $l = 4$ partial wave phase shifts, it seriously overbinds the three-α system as shown in (table 1) . The experimental binding energy of the three-α (^{12}C) system is $E_{3\alpha}$ = -7.275 MeV.

Table 1

L = 0 three- binding energy as a function of subsystem angular momentum l_{max} in case if fish-bone potential of Kircher and Schmid (FB-1)[9] and the results of Papp Z. and Moszkowski S. (FB-2).

L_{max}	FB-1	FB-2
2	0.057	-0.313
4	-15.47	-7.112
6	-15.63	-7.273
8	-15.63	-7.275

One may conclude that there is a need for three-body potential. This was the choice Oryu and Kamada[10]. They added a phenomenological three-body potential to the fish-bone potential of Kircher and Schmid[9] and found that a huge three-body potential is needed to reproduce the experimental data. But, Faddeev calculations[8] reveal that the $\ell = 4$ partial wave is very important to the three- α binding and, for this partial wave, the fit to experimental data is not so stellar. So Papp Z.and Moszkowski S.[8] concluded, that it may be possible to improve the agreement in the $\ell = 4$ partial wave and achieve a better description for the three- α binding energy in the point view of the these authors.

Thus as a local potential, two Gaussians plus screened Coulomb potential are added to form Fish-Bone II(FB-2):

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$$V_l(r) = v_1 \exp(-\beta_1 r^2) + v_2 \exp(-\beta_2 r^2) + \frac{4e^2}{r} erf(\sqrt{\frac{2a}{3}} r) \tag{9}$$

where v_1, β_1, v_2, β_2 and a are fitting parameters. In the fitting procedure Papp Z. and Moszkowski S. incorporated the famous ^{8}Be, l = 0 resonance state at E_{2b}^{exp} = (0.0916 -0.000003i) MeV, the ^{12}C three- α ground state energy E_{3b}^{exp} = -7.275 MeV, and the l = 0, l = 2 and l = 4 low energy phase shifts. With parameters v_1 = -120.30683493 MeV, β_1 = 0.20206127 fm^{-2}, v_2 = 49.06187648 MeV, β_2 = 0.76601097 fm^{-2} and a = 0.64874009 fm^{-2} , they achieved a perfect fit.

For the l = 0 two-body resonance state they get E_{2b} = 0.09161 – 0.00000303i MeV, and for the three-body ground state E_{3b} = -7.27502 MeV. Notice that unlike with the Ali-Bodmer potential, they achieved this agreement by using the same potential in all partial waves. Having this new α-α fish- bone potential from the fitting procedure, they also calculated the first excited state of the three- system. This state is a resonant state, and we got $E_{3\alpha}^{res}$ = (0.54- 0.0005i) MeV, which is again very close to the experimental value.

Solutions in a Three and Four Dimensional Space :

We consider solutions in a three dimensional variational space spanned by the orthonormal states l 1> , l 2 > and l 3> . In this case, we have twenty-one different symmetrized two-body matrix elements.

A HF-orbital λ will have the general form

$$|\lambda\rangle = \sum_{j=1}^{3} m_j^{\lambda} |j\rangle \tag{10}$$

Where

$$\sum_j m_j^{*\lambda} m_j^{\lambda'} = \delta_{\lambda\lambda'} \qquad , \sum_\lambda m_j^{*\lambda} m_{j'}^{\lambda} = \delta_{jj'} \tag{11}$$

And assume that the coefficient m_j 's are real.

To investigate the HF-solutions , we have to specify the alpha-alpha potential. The specific combinations chosen as the basic states depend on the symmetry of the intrinsic structure that is expected from the molecular alpha-particle model.

Now we chose basic states which are invariant with respect to the transformation of the symmetry group T_d [11]. Therefore, we chose our three basic states as

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$$|1\rangle = |0s0\rangle \tag{12}$$

$$|2\rangle = \frac{1}{\sqrt{2}}\{|0f2\rangle - |0f-2\rangle\} \tag{13}$$

$$|3\rangle = \frac{1}{\sqrt{\eta^2+2}}\{\eta|0g0\rangle + |og4\rangle + |0g-4\rangle\} \tag{14}$$

Where η is a parameter determined from the T_d symmetry.

We chose also basic states which are invariant with respect to the transformation of the symmetry group D_{3h} . Therefore, we chose our four basic states as

$$|1\rangle = |0s0\rangle \tag{15}$$

$$|2\rangle = \frac{1}{\sqrt{1+c^2}}\{|1s0\rangle + c|0d0\rangle\} \tag{16}$$

$$|3\rangle = \frac{1}{2}\{|0f3\rangle + |of-3\rangle\} \tag{17}$$

$$|4\rangle = |0g0\rangle \tag{18}$$

Here the oscillator shell model wave functions are given by

$$|\vec{r}, nlm\rangle = R_{nl}(r)Y_{lm}(\vartheta, \varphi) \tag{19}$$

$$\begin{aligned} R_{nl}(r) = [2\Gamma(n+1)/\beta_0^3\Gamma(n+l+3/2)]^{1/2}(r/\beta_0)^l \times \\ L_n^{l+1/2}(r^2/\beta_0^2)\exp(-r^2/2\beta_0^2) \end{aligned} \tag{20}$$

$\beta_0=(h/m_\alpha\omega)^{1/2}$ is the oscillator parameter.

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Nuclear Density and Form Factor :

For a system of number of alpha particles in the above basic states, the corresponding charge density normalized to unity, is readily found to be

$$\rho(r) = \sum_{lm} \rho_{lm}(r) Y_{lm}(\vartheta, \varphi) \tag{21}$$

The form factor corresponding to the spherical part of the charge density can be readily evaluated, one readily obtains

$$F(q) = \int \rho(r) e^{i\vec{q}.\vec{r}} dr \tag{22}$$

where q is the momentum transfer. The expression of F(q) has to be multiplied by a form factor F_α which account for the α-particle distribution where

$$F_\alpha(q) = e^{-q^2\beta_\alpha^2} \tag{23}$$

Fish-Bone Potential I Calculations :

Using the Fish-Bone potential I (FB-1) according to its equation (8), we have the following results of the binding energies and root mean square radii of ^{20}Ne nuclei, where the alpha particles are arranged according to the T_d and D_{3h} symmetries.

Table 2
The parameters used in our calculations of fishbone Potential I of ^{20}Ne

Symmetry	V_0(MeV)	β_1(fm^{-2})	a (fm^{-2})
T_d	-9	0.1889	0.55
D_{3h}	-19	0.1889	0.55

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Table.3

The calculated binding energies and root mean square radii of ^{20}Ne using the Fish-Bone potential I

Symmetry	Theor. B.E.(MeV)	Exp.B.E.(MeV)	Theor. rms (fm)	Exp. Rms (fm)[(a)]
T_d	-19.397	-19.18	2.711	2.91±0.05
D_{3h}	-20.454	-19.18	2.833	2.91±0.05

Fish-Bone Potential II Calculations :

Using the Fish-Bone potential II (FB-1) as in equation(9), we have the following results of the binding energies, root mean square radii and form factors

Table.4

The parameters used in our calculations of Fish-Bone Potential II of ^{20}Ne

Symmetry	V_1(MeV)	V_2(MeV)	β_1(fm^{-2})	β_2(fm^{-2})	a (fm^{-2})
T_d	-97	39	0.2021	0.7660	0.6487
D_{3h}	-65	21	0.2021	0.7660	0.6487

Table.5

The calculated binding energies and root mean square radii of ^{20}Ne using the Fish-Bone potential II

Symmetry	Theor. B.E.(MeV)	Exp.B.E.(MeV)	Theor. rms (fm)	Exp. Rms (fm)
T_d	-23.385	-19.18	2.963	2.91±0.05
D_{3h}	-20.368	-19.18	2.847	2.91±0.05

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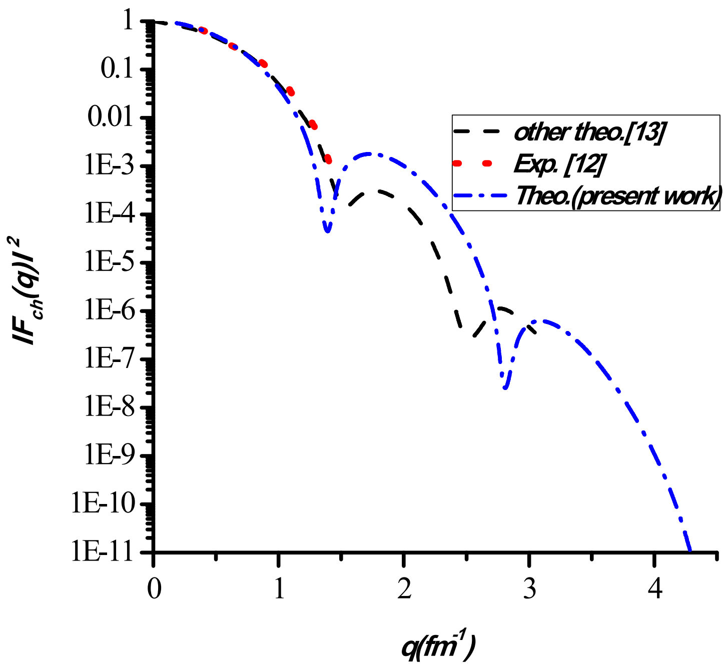

Fig. 1 : The form factor of ^{20}Ne which have the configuration symmetry T_d and using the parameters of Table 4 of FB-II .The points represent experimental values.

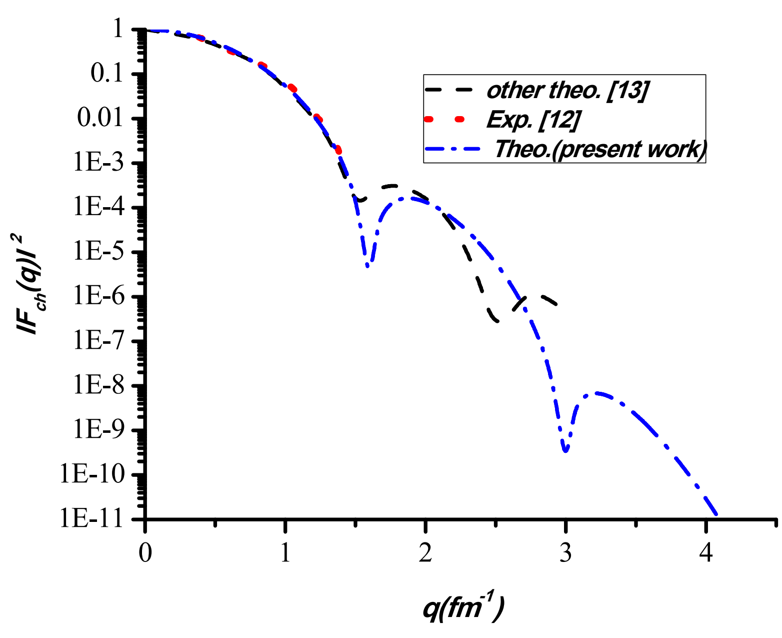

Fig. 2: The form factor of ^{20}Ne which have the configuration symmetry D_{3h} and using the parameters of Table 2 of FB-I. The points represent the experimental values.

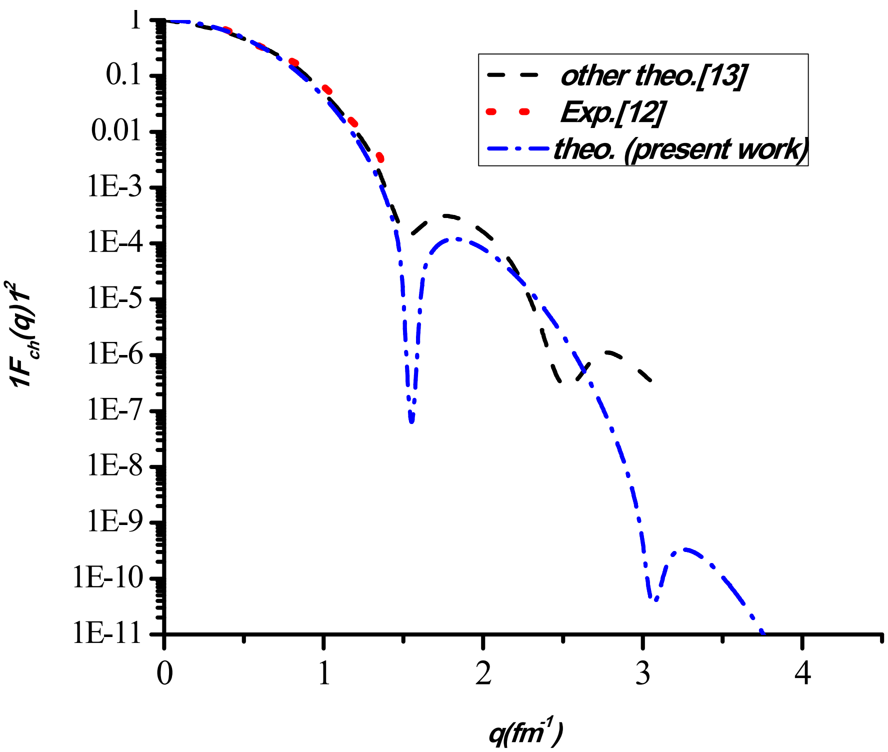

Fig. 3: The form factor of ^{20}Ne which have the configuration symmetry D_{3h} and using the parameters of Table 4 of FB-II. The points represent the experimental values.

III. DISCUSSION

By using the two types of the Fish-Bone potentials I and II and with the oscillator parameter B_t for ^{20}Ne in addition to the parameter of the alpha particle B_α ,and also considering the different configuration symmetries of ^{20}Ne; T_d and D_{3h} symmetries, we have got the nuclear structure properties of the ^{20}Ne nuclei such as binding energies, root mean square radii and form factors, and comparing the results with the experimental elastic scattering charge form factor [12] and other theoretical ones[13].

In case of Fish –Bone potential I (FB-1) , the binding energy of ^{20}Ne nuclei and the root mean square radius are in good agreement with the experimental results as shown in Table (3)**,** but we have not got form factors for ^{20}Ne nuclei in case of considering that the ^{20}Ne nucleus posses the configuration symmetry T_d . On the other hand, when we consider the four dimensional functional space, we have got the form factor as shown in Fig. (1). Applying the Fish-Bone potential II, we have got the binding energy, the root mean square radius and the form factors for ^{20}Ne nuclei either we consider the ^{20}Ne nuclei have T_d symmetry or D_{3h} symmetry as shown in Figs.(2) and Fig. (3) .

We conclude that the configuration symmetry D_{3h} of ^{20}Ne nucleus yielded better results than that of T_d symmetry, and the results are in acceptable agreement with the experimental points in addition to other theoretical ones.

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